Discrete hierarchical organization of social group sizes

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The 'social brain hypothesis' for the evolution of large brains in primates has led to evidence for the coevolution of neocortical size and social group sizes, suggesting that there is a cognitive constraint on group size that depends, in some way, on the volume of neural material available for processing and synthesizing information on social relationships. More recently, work on both human and non-human primates has suggested that social groups are often hierarchically structured. We combine data on human grouping patterns in a comprehensive and systematic study. Using fractal analysis, we identify, with high statistical confidence, a discrete hierarchy of group sizes with a preferred scaling ratio close to three: rather than a single or a continuous spectrum of group sizes, humans spontaneously form groups of preferred sizes organized in a geometrical series approximating 3–5, 9–15, 30–45, etc. Such discrete scale invariance could be related to that identified in signatures of herding behaviour in financial markets and might reflect a hierarchical processing of social nearness by human brains.

Keywords: social brain hypothesis; social group size; log-periodicity; fractal analysis

1. INTRODUCTION

Attempts to understand the grouping patterns of humans have a long history in both sociology (Coleman 1964) and social anthropology (Kottak 1991; Scupin 1992). While these approaches have been largely sociological in focus, attempts to understand grouping patterns in non-human primates have had a largely ecological focus (see Dunbar 1988). However, there has been recent interest in the extent to which group size and grouping patterns in primates might be constrained by cognitive factors (Dunbar 1992, 1998). The latter interests arise out of what has become known as the 'social brain hypothesis'.

The social brain hypothesis (Byrne & Whiten 1988; Barton & Dunbar 1997) argues that the evolution of primate brains was driven by the need to coordinate and manage increasingly large social groups. Since the stability of these groupings is based on intimate knowledge of other individuals and the ability to use this knowledge to manage social relationships effectively, the computational capacity of the brain (presumed to be broadly a function of its size) is assumed to impose a species-specific limit on group size. Attempts to increase group size beyond this threshold must inevitably result in reduced social stability and, ultimately, group fission. Dunbar (1992, 1998; Joffe & Dunbar 1997; also Sawaguchi & Kudo 1990) showed that there is a log-

linear relationship between social group size and relative neocortex volume in primates, and argued that this relationship reflected the computational capacity that any given species could bring to bear on its social relationships. Extrapolating these findings to humans led to the prediction that humans had a cognitive limit of approximately 150 on the average number of individuals with whom coherent personal relationships could be maintained (Dunbar 1993). Evidence to support this prediction has come from a number of ethnographic and sociological sources (Dunbar 1993). The fact that these relationships are not simply a matter of memory for individuals but, rather, of integrating and managing information about the constantly changing relationships between individuals within a group, is indicated by the fact that relative neocortex size correlates with a number of core aspects of social behaviour and socialization in primates (Byrne 1995; Pawlowski *et al.* 1998; Joffe 1997; Lewis 2000; Byrne & Corp 2004).

It has, however, always been recognized that both human and non-human primate groups are internally highly structured (e.g. Dunbar 1988). Further analyses (Kudo & Dunbar 2001) have indicated that at least one level of structuring (the grooming clique) also correlates with neocortex size. While the significance of these tiered groupings is not always apparent, there is strong *prima facie* evidence to suggest that human social groups (like those of other primates) consist of a series of sub-groupings

arranged in a hierarchically inclusive sequence (Hill & Dunbar 2003).

In this sequence, the core social grouping is the support clique, defined as the set of individuals from whom the respondent would seek personal advice or help in times of severe emotional and financial distress; its mean size is typically 3–5 individuals (Dunbar & Spoor 1995). Above this may be discerned a grouping of 12–20 individuals (often referred to as a sympathy group) that characteristically consists of all the individuals with whom one has special ties; these individuals are typically contacted at least once per month (Dunbar & Spoor 1995; Hill & Dunbar 2003). The ethnographic data on hunter-gatherer societies (summarized in Dunbar 1993) point to a grouping of 30–50 individuals as the typical size of overnight camps (sometimes referred to as bands); these groupings are often unstable, but their membership is always drawn from the same set of individuals, who typically number *ca*. 150 individuals. This last grouping is often identified in small-scale traditional societies as the clan or regional group. Beyond these, at least two larger-scale groupings have been identified in the ethnographic literature: the megaband of *ca*. 500 individuals and the tribe (a linguistic unit, commonly of 1000–2000 individuals) (Dunbar 1993).

In this paper, we provide the first systematic analysis of human grouping patterns, using data collated from the literature. Using spectral analysis, we show that there is a consistent pattern in the size of these groupings and, more importantly, that successive groupings in the hierarchy have a constant ratio.

2. MATERIAL AND METHODS

There is no universally accepted procedure for analysing human social groups, and all methods attempting to identify group sizes suffer from at least some sources of bias (small sample size, large inter-individual variability or differences in the criteria used to include individuals). Our strategy is to include all reasonable data and attempt to extract useful signals above the noise level by a careful analysis of the global dataset. We therefore searched the sociological and other literatures for quantitative data on social group and social network sizes in humans. For these purposes, we sought studies that provided quantitative data on the size of individuals' social networks, irrespective of how the social network itself was defined.

Most such studies focus on a particular kind of network (among those defined above in § 1). In addition to the data listed in Dunbar (1993), Dunbar & Spoor (1995) and Kudo & Dunbar (2001), we add the following data. The USA 1998 General Social Survey reports a mean size of 3.3 for support cliques in the USA (Marsden 2003). The mean sizes of sympathy groups are reported by Buys (1992) to be 14.0 in Egypt, 15.1 in Malaysia, 13.5 in Mexico, 13.8 in South Africa and 10.2 in the USA (Latkin *et al.* 1995). In separate samples in The Netherlands, they were reported to be 15.0 in 1995 (Kef 1997; Kef *et al.* 2000), 15.0 in 1992, 14.3 in 1992–1993, 14.8 in 1995–1996 and 14.2 in 1998–1999 (van Tilburg & van Groenou 2002), finally, Adams *et al.* (2002) reported them to be 14.4 in Mali (West Africa). Although a number of these studies have been carried out in the same country, we have considered each study to be an independent sample since they involve different datasets; nevertheless, averaging The Netherlands samples and treating them as a single data point does not alter the conclusions drawn.
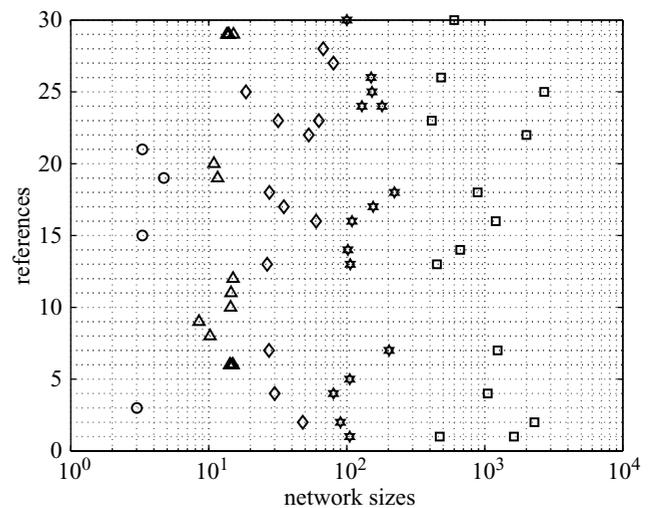
Figure 1. Presentation of our dataset of 61 group sizes. The ordinate is an arbitrary ordering of data sources and the abscissa gives the group sizes reported in each source. The symbols refer to the classification used in each of the studies: circles (support cliques), triangles (sympathy groups), diamonds (bands), stars (cognitive groups), and squares (small and large tribes). This classification is not used in our systematic analysis summarized in the other figures, to avoid any bias.

Only one study sought to estimate the size of successive social groupings for individual subjects (Hill & Dunbar 2003). These data were obtained from an analysis of Christmas card distribution lists, in which 42 UK-domiciled subjects logged the identities of all individuals in the households to which cards were sent and their relationships to these individuals. Participants were asked both to list everyone in the household to which they were sending a card and to state the quality of their relationship with each individual (using two metrics: how often they contacted the individual, and the emotional intensity of the relationship scored on a 0–10 Likert-type scale: for details, see Hill & Dunbar (2003)). Because this study uniquely provides data on the different grouping levels of which any one individual is a member, we treat these data separately from the census data obtained from the literature search.

3. RESULTS

We begin by analysing the data on groupings reported in the social networks literature. (The Christmas card distribution data will be dealt with separately: see below.) Figure 1 plots the sizes of the different grouping levels identified in the various studies.

We begin with a qualitative analysis of the data in figure 1, using the groupings that have conventionally been defined (see § 1). First, we denote S_1 as the mean support clique size, S_2 the mean sympathy group size, S_3 the mean band size, S_4 the mean community group size, and S_5 and S_6 the mean sizes of mega-bands and large tribes, respectively. Averaging across these grouping levels, the data give mean values of $S_0 = 1$ (individual or ego), $S_1 = 4.6$, $S_2 = 14.3$, $S_3 = 42.6$, $S_4 = 132.5$, $S_5 = 566.6$ and $S_6 = 1728$. To determine the possible existence of a discrete hierarchy, we construct the series of ratios S_i/S_{i-1}
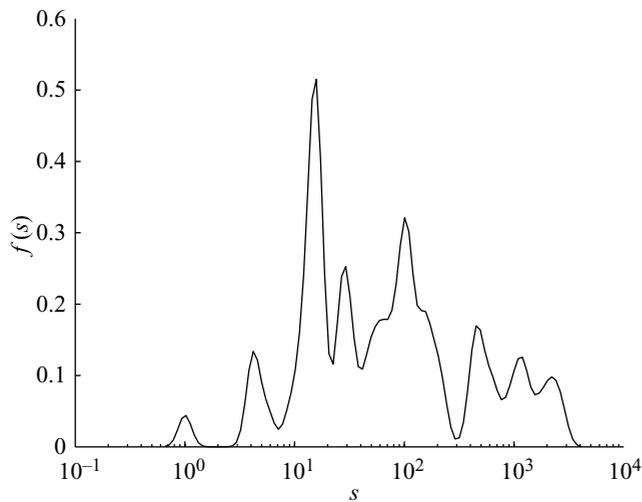
Figure 2. Probability density function $f(s)$ of size s estimated with a Gaussian kernel estimator in the variable $\ln(s)$ with a bandwidth $h = 0.14$. Varying h by 100% does not change $f(s)$ significantly.

of successive mean sizes:

$$S_i/S_{i-1} = 4.58, 3.12, 2.98, 3.11, 4.28, 3.05,$$
$$\text{for } i = 1,\ldots,6. \quad (3.1)$$

This suggests that humans form groups according to a discrete hierarchy with a preferred scaling ratio between 3 and 4 (the mean of S_i/S_{i-1} is 3.52).

To avoid any biases that might be present in the published census data, we next undertake a more systematic analysis that uses all the available data rather than just their means. The sample in figure 1 has 61 grouping clusters (including the ego) with estimates of mean size s_i available for $i = 1,2,\ldots,61$ clusters. We consider this sample to be a realization of a distribution whose sample estimation can be written as:

$$f(s) = \sum_{i=1}^{61} \delta(s - s_i), \quad (3.2)$$

where δ is Dirac's delta function. Figure 2 shows the probability density function $f(s)$ obtained by applying a Gaussian kernel estimation approach (Silverman 1986).

Our challenge is to extract a possible periodicity in this function in the $\ln(s)$ variable, if any. If the grouping clusters form a series of harmonics, the harmonics will have a constant ratio, and we would expect a periodic oscillation of $f(s)$ expressed in the variable $\ln(s)$ (known as its 'log-periodicity'; Sornette 1998).

Standard spectral analysis applied to $f(s)$ is dominated by the trend seen in figure 2, with a peak at a very low log-frequency corresponding to the whole range of the group sizes. We thus turn to generalized q-analysis or (H, q)-analysis (Zhou & Sornette 2002a), which has been shown to be very sensitive and efficient for such tasks. The q-analysis is a natural tool to describe discrete scale invariance (DSI) in fractals and multifractals (Erzan 1997; Erzan & Eckmann 1997). The (H, q)-analysis consists in constructing the (H, q)-derivative

$$D_q^H f(s) = \frac{f(s) - f(qs)}{[(1 - q)s]^H}. \quad (3.3)$$
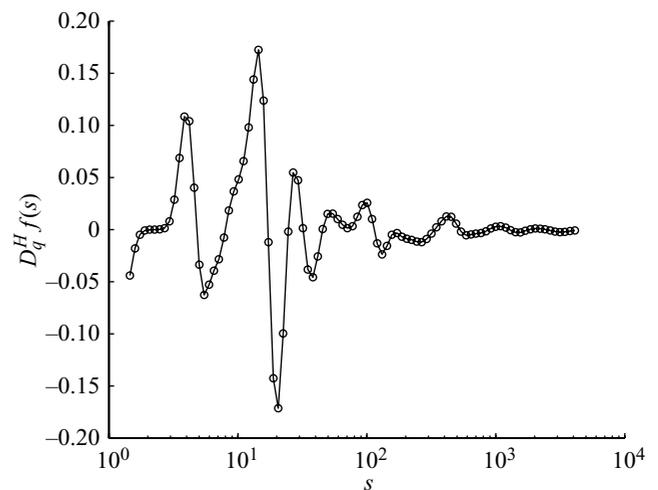
Figure 3. Typical (H, q)-derivative $D_q^H f(s)$ of the probability density $f(s)$ as a function of size s with $H = 0.5$ and $q = 0.8$.

Introducing an exponent H different from 1 allows us to detrend $f(s)$ in an adaptive way (that is, detrend it with different values of $[(1-q)s]^H$ at different s values). Note that the limit $H = 1$ and $q \to 1$ retrieves the standard definition of the derivative of f. A value of q strictly less than 1 makes it possible to enhance possible discrete scale structures in the data. To keep a good resolution, we work with $0.65 \leqslant q \leqslant 0.95$, because smaller values of q require more data for small values of s. To put more weight on the small group sizes (which are probably more reliable since they are obtained by conducting general surveys in larger representative populations), we use $0.5 \leqslant H \leqslant 0.9$. A typical (H, q)-derivative with $H = 0.5$ and $q = 0.8$ is illustrated in a semi-log plot in figure 3.

We then use a Lomb periodogram analysis (Press *et al.* 1996) to extract the log-periodicity in $f(s)$. Figure 4 presents the normalized Lomb periodograms of $D_q^H f(s)$ for different pairs of (H, q) with $0.5 \leqslant H \leqslant 0.9$ and $0.65 \leqslant q \leqslant 0.95$. This figure illustrates the robustness of our result. For the specific values $H = 0.5$ and $q = 0.8$ shown in figure 4, the highest peak is at $\omega_1 = 5.40$ with height $P_N = 8.67$. The preferred scaling ratio is thus $\lambda = \exp(2\pi/\omega_1) \approx 3.2$. The confidence level is 0.993 under the null hypothesis of white noise (Press *et al.* 1996). If the underlying noise decorating the log-periodic structure is correlated with a Hurst index of 0.6, the confidence level decreases to 0.99; if the Hurst index is 0.7 (which corresponds to an unreasonably large noise correlation), the confidence level falls to 0.85 (Zhou & Sornette 2002b).

The Lomb periodograms also exhibit a second peak at $\omega_2 = 9.80$ with height $P_N = 5.48$. This can be interpreted as the second harmonic component $\omega_2 \approx 2\omega_1$ of the fundamental component at $\omega_1 = 5.40$. The amplitude ratio of the fundamental and the harmonic is 1.26. The coexistence of the two peaks at ω_1 and $\omega_2 \approx 2\omega_1$ strengthens the statistical significance of a log-periodic structure. To see this, we constructed 10^4 synthetic sets of 61 values uniformly distributed in the variable $\ln(s)$ within the interval $[0, \ln(2000)]$. By construction, these 10^4 sets, which are exactly of the same size as our data and span the same interval, do not have log-periodicity and thus have no characteristic sizes. We then applied the same procedure as
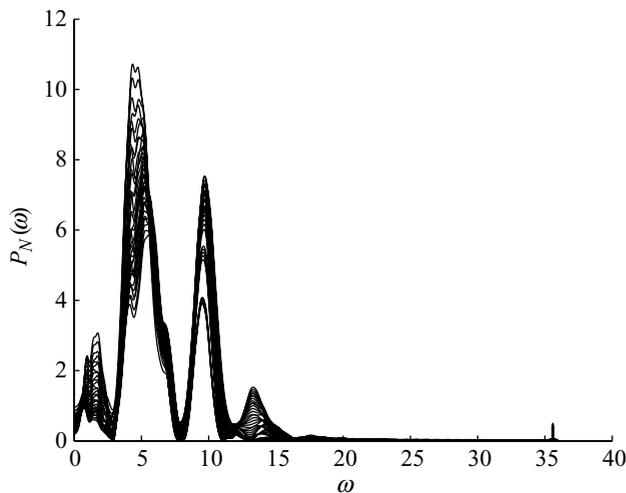
Figure 4. Normalized Lomb periodograms $P_N(\omega)$ as a function of angular log-frequency ω of the (H, q)-derivative $D_q^H f(s)$ for different pairs of (H, q) with $0.5 \leqslant H \leqslant 0.9$ and $0.65 \leqslant q \leqslant 0.95$.
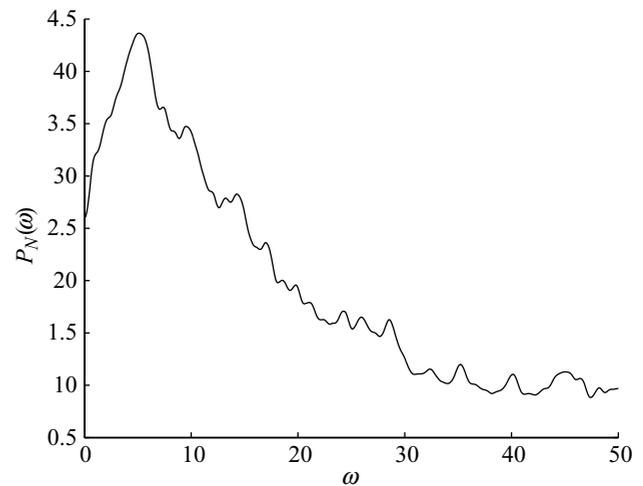

Figure 5. Average Lomb periodogram $P_N(\omega)$ of the (H, q)-derivative $D_q^H f(s)$ with respect to the number of receivers of the residual contact frequency for each individual in the Christmas card experiment, as a function of the angular log-frequency ω of the (H, q)-derivative, over the 42 individuals and different pairs of (H, q) with $-1 \leqslant H \leqslant 1$ and $0.80 \leqslant q \leqslant 0.95$.

for the real dataset to these synthetic datasets and obtained 10^4 corresponding Lomb periodograms. Finally, we performed the following tests on these Lomb periodograms: find the highest Lomb peak (ω, P_N). If $P_N > 8.5$, check if there is at least another peak at $2\omega \pm 1$ with its P_N larger than 5.5. A total of 238 sets among the 10^4 passed the test, suggesting a probability that our signal results from chance equal to 0.024. The probability that there are at least two peaks (one in $4.9 < \omega < 5.9$ with $P_N > 8.5$ and the other in $9.5 < \omega < 11.5$ with $P_N > 5.5$) is found equal to $77/10^4$, giving another estimation of 0.992 for the statistical confidence of our results.

Another metric consists in quantifying the area below the significant peaks found in the Lomb periodogram of our data and comparing them with those in the synthetic sets. We count the area of the main peak of the Lomb periodogram at ω and add to it the areas of its harmonics whose local maxima fall in the intervals $[(k - (1/5))\omega, (k + (1/5))\omega]$ for $k = 2, 3, \ldots$, around all its harmonics. The area associated with a peak is defined as the region around a local maximum delimited by the two closest local minima bracketing it. The fraction of synthetic sets which give an area thus defined larger than the value found for the real data is 6–7%, depending on the specific values H and q used in the analysis.

We applied the same analysis to individual social networks based upon the exchange of Christmas cards (Hill & Dunbar 2003). This study indicated that contemporary social networks might be differentiated based on the frequency of contact between individuals, but that both 'passive' and 'active' factors may determine contact frequency. Controlling for the passive factors (distance apart, and whether the contact was overseas or a work colleague) allowed the hierarchical network structure to be examined based on the residual (active) contact frequency. Starting from the residual contact frequencies, we constructed their (H, q)-derivative with respect to the number of people contacted for each individual, obtained the Lomb spectrum of the (H, q)-derivative and then averaged them over the 42 individuals in the sample (figure 5). The

very strong peak at $\omega = 5.2$ is consistent with the previous results with a preferred scaling ratio from the expression $\lambda = \exp(2\pi/\omega_1) \approx 3.3$ (Sornette 1998) for the smaller grouping levels in this study (i.e. group sizes below 150).

In summary, all these tests suggest that the evidence in support of our hypothesis is significantly unlikely to result from chance, but rather reflects the fact that human group sizes are naturally structured into a discrete hierarchy with a preferred scaling ratio close to 3.

4. DISCUSSION

Collating a variety of measures collected under a wide range of conditions and in different countries, we have documented a coherent set of characteristic group sizes organized according to a geometric series with a preferred scaling ratio close to three. The fact that the signature of this scaling ratio comes through so strongly despite the fact that the data derive from a variety of different small- and large-scale societies suggests that it is very much a universal feature. Were it the case that scaling ratios differed between societies, pooling data would have tended to obscure any relationships that might have been present.

Indeed, it turns out that similar hierarchies can be found in other types of human organizations, although the consistency of the patterning has not previously attracted comment. Of these, the military probably provides the best examples. In the land armies of many countries, one typically finds sections (or squads) of *ca.* 10–15 soldiers, platoons (of three sections, *ca.* 35), companies (3–4 platoons, *ca.* 120–150), battalions (usually 3–4 companies plus support units, *ca.* 550–800), regiments (or brigades) (usually three battalions, plus support; 2500+), divisions (usually three regiments) and corps (2–3 divisions). This gives a series with a multiplying factor from one level to the next close to three. Could it be that the army's structures have evolved to mimic the natural hierarchical groupings of everyday social structures, thereby optimizing the cognitive processing of within-group interactions?

Stock market behaviour provides another example of the same kind of phenomenon (and one that we happen to have investigated). The existence of a discrete hierarchy of group sizes may provide a key ingredient in rationalizing the reported existence of DSI in financial time series in so-called 'bubble' regimes characterized by strong herding behaviours between investors (Sornette 2003). Johansen *et al.* (1999, 2000) have proposed a model to explain the observed DSI in stock market prices as resulting from a discrete hierarchy in the interactions between investors. Recent analyses of DSI in market regimes with a strong herding component have also identified the presence of a strong harmonic at 2ω, similar to the findings reported here (Johansen & Sornette 1999; Sornette & Zhou 2002). Strong herding behaviour occurs only when groups of investors coordinate their buy and sell orders; the coordinated buy and sell orders that occur during a strong herding market phase thus expresses, better than at any other time, the natural inner structure of the community of traders. By contrast, herding is absent when investors disagree on what will be the next market move; as a result, the aggregate market orders do not express the inner hierarchical structure of the community.

The fact that DSI is found only during stock market regimes associated with a strong herding behaviour suggests that it may reflect the fact that a discrete hierarchy of naturally occurring group sizes characterizes human interactions whether they be hunter-gatherers or traders. The findings reported here suggest that this discrete hierarchy may have its origins in the fundamental organization of any social structure and be deeply rooted within the cognitive processing abilities of human brains.

When dealing with discrete hierarchies, it may be important to distinguish between the specific group sizes and their successive ratios. It may be that the absolute values of the group sizes are less important than the ratios between successive group sizes. If the ratio of group sizes is interpreted as a fractal dimension (specifically, the ratio is related to the imaginary part of a fractal dimension: see Sornette (1998) and references therein), this would imply that, depending on the social context, the minimum 'nucleation' size (in the range 3–5 in previous examples) may vary, but the ratio (close to three) might be universal. The fundamental question, then, is to determine the origin of this discrete hierarchy. At present, there is no obvious reason why a ratio of three should be important.

Equally, however, we have little real understanding of what mechanisms might limit the nucleation point to a particular value. We do not even know, for example, whether the constraint is a cognitive one (e.g. memory for individual identities versus capacity to manage information about relationships); or a time budgeting one (how much time has to be invested in interaction with an individual to create a bond of a particular strength, and then how many such bonds can be fitted into a given time-scale). Nor do we know much about how larger-scale groupings are built up out of smaller ones. A hierarchical structure could, for example, be built up by each individual interacting with, say, three new individuals in an expanding network, or it might be the result of rather discrete small subgroups held together through a subset of individuals who act as 'weak links' in the small-worlds sense—although there is some evidence for the latter in respect of both primate social

groups (Kudo & Dunbar 2001) and at least some aspects of human behaviour (Stiller *et al.* 2004). Considerable additional work will need to be done on both these components if we are to understand why these constraints on human grouping patterns exist and exactly what their significance might be.

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